AN EFFICIENT OPTIMAL-POWER-FLOW SOLUTION VIA IMPERIALIST COMPETITIVE ALGORITHM

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Abstract. This paper presents an Imperialist Competitive Algorithm (ICA) for Optimal Power Flow (OPF) solution. ICA procures an efficient modeling of non-differentiable and non-linear objective and constraints in OPF optimization problem. Simple implementation, fast convergence within a scant number of steps, and a slimmer objective value are parts of the proposed ICA-OPF algorithm. As a result, ICA-OPF is enabled handling more realistic systems. To evaluate the proposed algorithm, simulations are also conducted on two universally-appreciated metaheuristic techniques of Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). Obtained results show that the developed ICA-OPF outruns the GA and PSO both in terms of CPU clocking and objective value. The IEEE 57-bus system is employed to test the proposed algorithm against conventional techniques. Based on the simulation results, the proposed method can be installed in Load Dispatch Center with a better solution quality and extensible to larger-scale utility size problem.

Keywords: Imperialist Competitive Algorithm, Optimal Power Flow (OPF), Evolutionary Algorithm

چکیده: این مقاله یک الگوریتم رقابتی امپریالیستی (ICA) برای راه حل پخش توان بهینه (OPF) ارائه می دهد. ICA مدلسازی کارآمدی از اهداف و محدودیت‌های غیر قابل تمایز و غیرخطی را در مسئله بهینه‌سازی OPF تهیه می‌کند. پیاده سازی ساده، همگرایی سریع در تعداد کمی از مراحل، و مقدار هدف کوچک‌تر، بخش‌هایی از الگوریتم پیشنهادی ICA-OPF هستند. ICA-OPF قادر است سیستم‌های واقعی را مدیریت کند. برای ارزیابی الگوریتم پیشنهادی، شبیه‌سازی‌ها نیز بر روی دو تکنیک فراابتکاری که به طور جهانی ارائه شده‌اند و الگوریتم ژنتیک (GA) و بهینه‌سازی ازدحام ذرات (PSO) انجام می‌شوند. نتایج به‌دست‌آمده نشان می‌دهد که ICA-OPF توسعه‌یافته از GA و PSO هم از نظر سرعت و هم از نظر مقدار هدف، پیشی می‌گیرد. سیستم ۵۷ باس IEEE برای آزمایش الگوریتم پیشنهادی در برابر تکنیک‌های مرسوم استفاده شده است. بر اساس نتایج شبیه‌سازی، روش پیشنهادی می‌تواند در مرکز توزیع بار، راه‌حل بهتری ارائه دهد و مشکل اندازه سیستم را نیز برطرف کند.

کلمات کلیدی: الگوریتم رقابتی امپریالیستی، پخش توان بهینه (OPF)، الگوریتم تکاملی

1-Introduction

With advent of restructuring in electric industry, hourly power blocks are treated as a transactional commodity. It created a market with a number of players promoting better efficiency, higher quality, and reliable power served to the customer at a competitive price. Therefore, the legacy operation and control schemes are superannuated. They have been either overhauled or partly retrofitted so as to accommodate new forms of business-oriented operational aspects throughout the three levels of generation, transmission and distribution [1].

To meet the load demand at a geographically wide range in the country, System Operator (SO) must efficiently dispatch the available generations of the on-line units in the system. It is imperative for SO to ensure that the generation adjustments are satisfying inter-temporal and spatial constraints in system [1], [2]. Likewise it requires retaining a secure margin for voltage profiles as well as the dynamic response for such a complex system that embraces a plethora of constraints into the model. To end this, Optimal-Power-Flow (OPF) algorithm has been introduced to optimally allocate generation levels of the committed units at the premise of satisfying unit-wise and system-wise active and reactive constraints. OPF builds an optimization model where the objective is production (fuel) cost minimization subject to the operational constraints and bounds [2], [3].

OPF is the second step from a sequence of steps is taken to optimize operation in power system namely, Unit Commitment (UC), OPF, and Load Flow (LF). In this sense, UC can be addressed as a pre-

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dispatch start-up scheduling process that procures the most money-savings in this context. OPF is a real-time optimization process and it addresses a cost-effective and secure system operation within an hourly service [2-4].

OPF constitutes a mathematical problem whose inputs are generator's quadratic production curves, matrix of constraints and bounds and its output will be the unit's output level and back-up reserve associated with each available unit. In another word, OPF rigorously appoints reliability and security margins of the system in a real-time regime [1-3].

To solve OPF problem, a number of methods were proposed and applied. Among them Linear Programming (LP), Interior Point Method (IPM), Sequential Quadratic Programming Method, Generalized Reduced Gradient Method, and Newton-Raphson Methods have found a great deal of attentions by utilities around the globe. These techniques offer a systematic and diversified solution which can be tractable at any instant of time. However, such classical method represented several drawbacks in real-world application [4], [5].

Deterministic methods embed a gradient operator in the solution mechanism, which may trap in a local minima, and end up with inefficient solution. The reason is, many practical constraints in the utilities are of non-linear nature that complicates solution process in classical techniques. To remedy this, approximations and heuristics are added to the classical methods as a way to enable the global-best convergence in a reasonable range of solution time [5-7].

In appose to classical methods, meta-heuristic techniques represented an outstanding performance especially in very non-linear systems of equations. In essence, the biologically-inspired algorithms procure a solution by mimic of their lifestyle and the way out-surviving in their habitat.

This paper attempts to solve OPF by means of one meta-heuristic technique to show the effectiveness of the method and simplicity of their implementation.

2- Modern Optimization Techniques

New series of artificial intelligent calculation tools are introduced to resolve the limitations of traditional optimization algorithms. Some of the dominant modern techniques include: Evolutionary Programming (EP), Genetic Algorithm (GA), Particle Swarm Optimization (PSO) artificial neural network, colonial competitive algorithm. Some of these methods that are newly developed imitate a certain natural phenomena in their quest for optimization, like development of species (EP,GA,PSO), or the neural system of human (ANN), or a social behavior (ICA). These techniques are successfully applicable in a wide variety of optimization problems in which holistic responses are more important than individual ones or when the issue includes non-differentiable and highly non-linear environment [8], [9].

Furthermore, these methods are well-known for the rapid search in a huge and ample search space. Additionally, they can handle a system with a pool of uncertainties in their operation framework [8].

3- Optimal Power Flow

OPF is made of an objective function, which is total production cost and a number of constraints. The first and foremost constraint is the power balance equation which ensures the adequacy of the supply at each operational time-interval [4], [7].

$$P_D = \sum_{i=1}^{n_g} P_{gen(i)} = \sum_{i=1}^{n_l} P_{load(i)} + P_L \tag{1}$$

where,

$P_{gen(i)}$: production power by i th generator ;

$P_{load(i)}$: consumption power by i th load ;

P_L : the lost power in transfer lines ;

n_g : the number of available generators in the system ;

n_i : number of loads in the system ;

P_D : the demand power.

Eq. (1) indicates that the total power generated must be equal to the load demand plus with transmission losses.

The capacity of each generator is limited to its maximum production. As of power stability prudence, the generator's output cannot be reduced to zero, but a certain minimum level if the unit was taken up. Therefore, there are two limitations of maximum and minimum for generators as in (2).

$$P_{gen(i),\min} \le P_{gen(i)} \le P_{gen(i),\max} \tag{2}$$

In eq. (1), represents the power losses in the lines. The power losses can be calculated with the help of Kron's formula.

$$P_L = \sum_{i=1}^{n_g} \sum_{j=1}^{n_g} P_i B_{ij} P_j + \sum_{i=1}^{n_g} B_{0i} P_i + B_{00} \tag{3}$$

Where is the total real power losses, and *Bij* are the loss coefficients or B-coefficients.

The total cost of production can be calculated as a sum of all the costs of power production in each generator. Therefore, the total cost of power production can be easily obtained using relation (4).

$$C_t = C_1 + C_2 + \cdots + C_{n_g} \tag{4}$$

The costs of i_{th} power plant C_i can be obtained from the fuel cost curve of the power plant. Fuel cost curves, for the purpose of simplification, is estimated as a quadratic function and is shown in (5). To have more accuracy it is possible to estimate this curve with higher order polynomials or with the help of multi-criteria functions that takes discontinuity effects into account.

$$F(P_i) = \sum_{i=1}^{n_g} \left(a_i + b_i P_i + c_i P_i^2\right) \quad \frac{\$}{hr} \tag{5}$$

In a relation that, a_i, b_i and c_i are fixed coefficients of the cost function, P_i is the production power of i_{th} generator.

A. Linear Programming

In mathematics, the linear programming (LP), is a technique for the optimization of the function of linear targets, with regards to linear equality and non-equal relations. The linear programming paves the way for the method obtaining the best results for a mathematical model [5], [11], [12].

$$f(x_1, x_2, \ldots, x_n) = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n + d \tag{6}$$

The linear programming finds a point in the above polynomial function that has the least (or most) value. Such point might not exist, but if it exists, the search through the summits of function borders guarantee that we find at least one of them.

$$Maximaze \quad c^T x \quad Subject\ to: \quad Ax \le b \tag{7}$$

Linear programming problems can be expressed in canonical form: x indicates the vector of variables (that should be determined), while b and c are the vectors of (determined) coefficients and A (determined) is the matrix of coefficients. The first phrase of the function is our target that should be maximum and the second phrase are our equations and constraints that draw the Canucks borders and the function should be optimized on these borders.

The linear programming can be applied in different fields. It is especially used in economic problems. However, it can be used for some engineering problems as well. Industries that may use linear programming include transportation industry, energy, telecommunication, and productive industries. Despite their great convergence properties and their increasing application, it has some drawbacks:

1-Convergence to general or specific solutions is highly dependent to the initial guess.

2-Each technique is suitable for one solution of specific load distribution on the basis of the mathematical nature of the target function or constraints.

3-They are developed on the basis of some theoretical assumptions like the assumption of being convex, differentiability, and continuity that might not be suitable in real conditions.

B. Imperialist Competitive Algorithm (ICA)

Similar to other evolutionary algorithms, this algorithm begins with a basic random population that each of them is called a “country”. Some of the best elements of the population (equal to the elites in the genetic algorithm) are selected as the imperialist. The rest of the population will be considered as the colonies. Colonialists, based on their power, draw these colonies towards themselves in a certain trend that will be discussed later. The power of each empire depends both on the imperialist country (as the central core) and its colonies. In the mathematical condition, this dependence is modeled with defining the empire power in form of the total power of the imperialist country plus a percentage of the average power of its colonies. By the emergence of the first empires, the imperialistic competitions begin among them. Each empire that cannot compete and add to its power (or at least prevent its weakening) will be eliminated from this colonial competition. Therefore, the survival of an empire depends on its power in capturing the colonies of the rival emperors. Consequently, in the imperialistic competitions, the stronger empires increase their power and the weaker ones will be eliminated. To increase their power empires will need to develop their colonies as well [7-9].

To start the algorithm, we create N number of the first countries; N number of the best members of this population (countries that have the least amount of cost function) will then be selected as the imperialists. The rest is N number of countries that consists the colonies that each belong to one empire. To divide the first colonies among the imperialists, we specify a number of colonies to each imperialist according to their power. To this end, with having the costs of all imperialists, we consider their normalized cost as follows:

$$C_n = \max\{c_i\} \tag{8}$$

In this formula, c_n is the cost of n^{th} imperialist, $\max\{c_i\}$ is the highest cost among empires, and c_n is normalized cost of this imperialist. An imperialist that has higher cost (weaker one) has lower normalized costs. With having normalized cost, the relative normalized power of each imperialist will be calculated as follows, and accordingly the colonies will be shared among the imperialists.

$$P_n = \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \tag{9}$$

Yet from another perspective, the normalized power of an imperialist is the ratio of colonies that are being managed by an imperialist. Therefore, the initial number of the colonies of an imperialist will be equal to:

$$N.C._n = round\,\{P_n.N_{col}\} \tag{10}$$

In which $N.C_n$ is the first number of the colonies of an empire, and N_{col} is the total number of colonies in the population of the first countries. *Round(.)* is also a function that gives the closest correct number to a decimal number. Considering the *N.C* for each empire, we select this number of basic colonies randomly and give it to the imperialist *n*, and with having the early conditions of all empires, the colonial competition algorithm will begin. The development trend is in one ring which is continued until it meets a suspension condition [5], [9].

The movement of colonies towards the imperialists (absorption or assimilation policy) was made with the purpose of solving the culture and social structure of colonies in the culture of the central government. The colonialist countries would increase their influence in their colonies by creating the infrastructure of transportation system, foundation of universities, and other infrastructure and construction businesses [8]. If in the move from the colony to the colonist their distance be equal to, the amount of movement of the colony towards the colonist X a uniform random value from zero and $d \times \beta$ will be selected. The value of β is between one and two. A good choice for β can be the number 2. Existence of a coefficient near 2 for β causes that the colony approach the colonist from different directions. Examination of the history of assimilation, there was an obvious fact the despite of pursuing their absorption policies the failed to reach all their goals, and divergence would be seen in the outcome. In the proposed algorithm, this probable algorithm is done by adding a random angel to the absorption trend of colonies. To this end, in the move of colonies towards colonists, some random angle is added to the direction of the movement of the colony. Therefore, instead of moving to the amount of X towards the colonist country and in the direction that leads to colonists, to the same amount, yet with the deviation θ moves in the direction. The value of θ is considered randomly and with the uniform distribution [10], [11].

$$X = U\left(0, \beta \times d\right) \tag{11}$$

In this relationship, γ is a desired parameter that its increase causes the increase of the search around the imperialist, and its reduction causes that colonies move at the proximity of the vector leading to the colonist as much as possible [5], [10].

C. Exchanging Positions of the Imperialist and a Colony

The policy of absorption and in the meantime destroying the social-political structures of the colonial country was to their advantage in some cases. Some of these countries gained self-confidence due to such policies and after some time became the leaders of their own countries to free their country from the grip of colonialism. In modeling this historical event in the proposed algorithm, it has been administered that it is possible that colonies in their move towards colonists obtain a better position than them. In this condition, the colonist and the colony change their positions and the algorithm will continue its existence with the colonialist in a new position. And this time, it is the new imperialist country that makes assimilation policies on its colonies [[9], [11].

D. The Total Power of an Empire

The power of an empire is equal to the power of a colonialist country plus some percentage of all of its colonies. Therefore, the total cost of an empire is equal to:

$$T.C._n = Coast\left(imperialist\right) + \xi \; mean\left\{Coast\left(colonies\ of\ \ empire\right)\right\} \tag{12}$$

In which is the overall cost of the empire and is usually considered between zero and one and close to zero. The small values of means that the overall cost of an empire is almost equal to the costs of its central government (imperialist country), and its increase (values of) causes the increase in the impact of the amount of costs of the colonies of an empire in determination of its overall costs. In a typical condition its 0.05 value has led to acceptable responses in most of the implementations.

In the imperialistic contest, each empire that fails to increase its power or loses its power will be eliminated gradually. For modeling this fact, suppose that the weakening empire is the weakest existing empire. So, in repeating the algorithm one or several of the weakest colonies take the weakest empire, and to capture these colonies, we create a contest among all the empires. The aforementioned colonies will not necessarily be captured by the strongest empires but the stronger empires are more possibilities to capture the colonies. First on the basis of the overall costs of the empire, we determine the overall normalized costs.

$$N.T.C._n = \max\left(T.C._i\right) - T.C._n \tag{13}$$

In this relationship, $T.C.._n$ is the overall cost of empire n and $N.T.C._n$ is the overall normalized cost of that empire. Each empire that contains low $T.C.._n$, has more $N.T.C._n$. In fact $T.C.._n$ is the overall cost of an empire and $N.T.C._n$ is the overall power of that country. The possibility of capturing the colony by each empire is calculated as follows:

$$Ppn = \frac{N.T.C.n}{\sum_{i=1}^{Nimp} N.T.C.i} \tag{14}$$

To divide the aforementioned colonies randomly, yet with the possibility dependent to the possibility of being captured by each empire among all empires, we specify the aforementioned colonies to an empire that its index in the below vector be bigger than others [6], [11].

$$D = [P_{Pi} - r_i]_{1 \times N_{imp}},\ r_i \sim U(0,1), i = 1, 2, \cdots, N_{imp} \tag{15}$$

4- Simulation Results

The proposed ICA-OPF method is tested on IEEE standard 57-Bus system. This case contains 80 transmission lines and 7 generators as shown in Figure.1.

Specifications of the generators are mentioned in Table I [4], in this table a, b, c are the constants of the quadratic cost function to procure more accuracy. p_{max} and p_{min} are the maximum and minimum of each generator. The maximum load demand of the system is 1250.8MW.

For the calculation of ICA algorithm parameters are defined as below:

Number of Countries: 800

Number of Imperialists: 22

Number of Colonies: 778

Number of Repetitions: 500

Percentage of the Revolution of Colonies: 0.3

Assimilation: 2

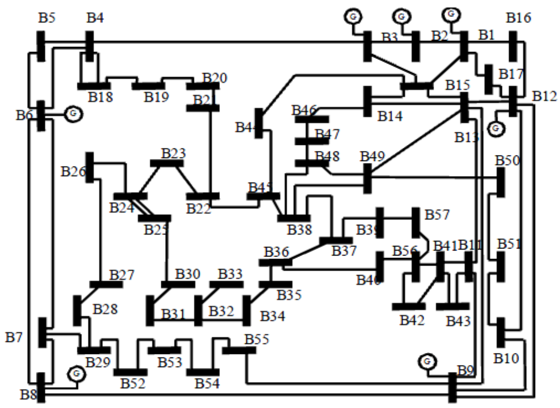

Fig.. 1 Single-line diagram of IEEE 57-bus test system.

The total number of iteration is set on 20 and the best results were obtained with the abovementioned parameters.

Table 1. THE SPECIFICATIONS OF THE GENERATORS

Generator	Bus	a	b	c	P_{min}	P_{max}
1	1	0.01	0.30	0.20	0.00	575.88
2	2	0.01	0.30	0.20	0.00	100.00
3	3	0.01	0.30	0.20	0.00	140.00
4	6	0.01	0.30	0.20	0.00	100.00
5	8	0.01	0.30	0.20	0.00	550.00
6	9	0.01	0.30	0.20	0.00	100.00
7	12	0.01	0.30	0.20	0.00	410.00

Optimization is done in three phases. In the first case, the transmission loss is ignored. In the second case, the transmission loss is considered constant and equal to 19.06 MW. In the third case the losses are treated as a variable loss.

Case1: the losses have been ignored. The results of proposed ICA-OPF and solution from GA-OPF and PSO-OPF algorithms are given in Table II. It is demonstrable from the results that the proposed ICA-OPF method outperforms other algorithms in terms of total operation cost (higher accuracy).

Table 2. COMPARISON OF RESULTS FOR CASE 1

Variable	PSO	GA	ICA
*PG*1 (MW)	267.31	262.02	262.12
*PG*2 (MW)	99.87	99.99	99.99
*PG*3 (MW)	141.23	140.02	140.14
*PG*6 (MW)	100	99.99	100.01
*PG*8 (MW)	272.39	276.03	275.78
*PG*9 (MW)	99.99	99.66	99.99
*PG*12 (MW)	270.01	272.76	272.68
Cost ($/hr)	3063.96	3063.07	3062.89

Case2: The losses line is set to a fixed value of 19.6 MW. According to the results shown in Figure.2 , the objective function is as low as $ 3173.982 for the proposed ICA-OPF technique. It is due to the prudent dispatch of 7 units to minimize the cost. For example, unit#7 which is the most expensive unit is much lower in ICA as compared with GA and PSO in this case.

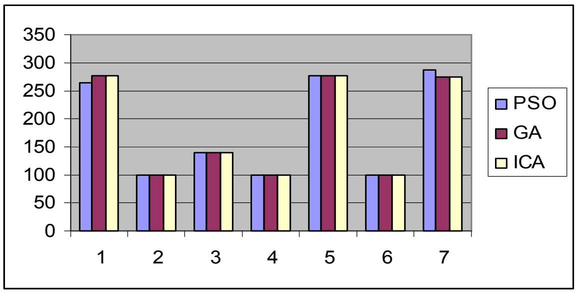

Fig. 2. The production of each generator with constant losses

Case3: In this section, line losses are considered to be variable. Optimization problem is solved and line losses provided in the last row of Table III.

Table 3. DISPATCH RESULTS FOR CASE 3

Variable	PSO	GA	ICA
PG1 (MW)	264.94	266.34	268.63
PG2 (MW)	100.02	99.99	100
PG3 (MW)	140	139.95	140
PG6 (MW)	99.98	100	100
PG8 (MW)	277.09	280.53	275.31
PG9 (MW)	100	100	100
PG12 (MW)	287.83	282.81	281.62
Cost ($/hr)	3176.38	3171.785	3158.09
PLoss (MW)	19.07	18.78	14.76

The losses associated with ICA-OPF algorithm is only14.76MW while it is higher in case of PSO and GA algorithm with 19.07 MW and 18.7 MW respectively. On the other hand, the devolved ICA-OPF outshines the other techniques by offering more power-save in transmission line and judicious dispatching of units in the system.

The ICA-OPF avoids higher production of the remote units and instead increase the output of the units near the load center which provide less transmission loss and lighter total operation cost.

Figure. 3 shows the convergence evolution of cost function or the case 3. As one can infer from Figure. 3, the proposed algorithm takes less number of steps (iteration) to hit the optimal solution. It will depend on the type of constraints in the model, either the equality or inequality constraints. In this sense if the repetitions limit relaxed (beyond 500), the cost function is further improved and converges to the better solution.

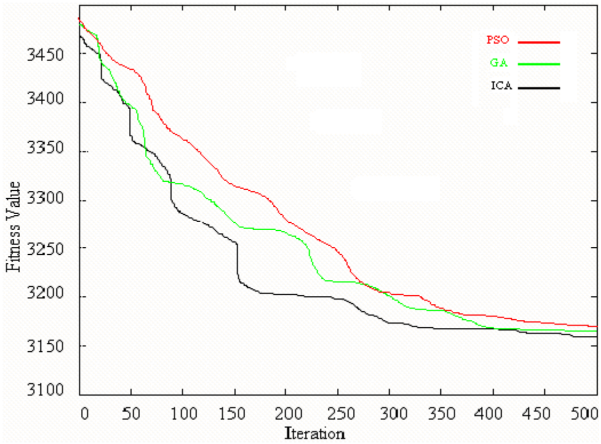

Fig. 3. cost function-repetition evolution progress

Figure. 4 exhibits generation levels of the generators in the proposed method ICA-OPF model which favorably juxtaposed with other methods for a quick comparison study. Units# 1,7, 5 contributed the most and units # 2, 4,6 kept in the lowest possible output so as to reduce the total costs. Nevertheless, more expensive generators are back off and cheaper units ramp up as much as satisfying load demand and system constraints.

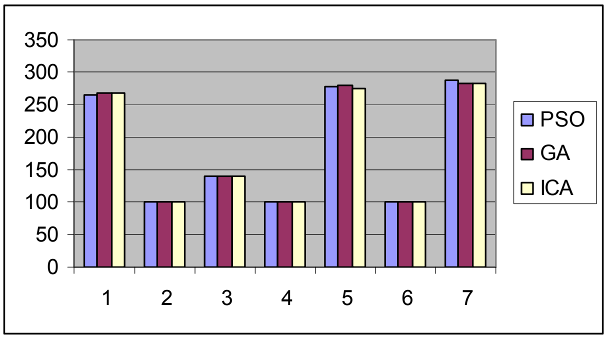

Fig. 4. Generator output for case 3

Execution time performance for each method is tabulated in Table IV. As one can observe, the proposed ICA-OPF outruns PSO and GA for all three case studies. For example, in case 3, the computational performance of ICA-OPF against the PSO and GA shows the speed-up factor of 1.13, 1.23 respectively.

Table 4. COMPARISON OF THE TIME PROGRAM EXECUTION

	GA	**PSO**	**ICA**
Case1	56s	54s	48s
Case2	61s	58s	51s
Case3	69s	61s	53s

5-Conclusions

In this paper, the Imperialist Competitive Algorithm (ICA) algorithm is enforced to solve Optimal Power Flow (OPF) problem. The proposed ICA-OPF algorithm outperforms in terms of accuracy and computing time of the solution. One of the main strength of the ICA is the fast convergence to the global optima as oppose to the classical methods where usually trapped in local minima, which led to an inefficient answer. Further, the randomness mechanism existed in ICA algorithm allowed modeling uncertainties, non-differential, and non-linear complex systems. As of simple implementation of ICA compared with GA and PSO, it offers fast convergence and better accuracy (optimal dispatch of units). The solution quality of the proposed ICA-OPF represents a promising horizon for larger scale systems with plethora of nonlinear constraints.

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